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PROTON FLUXES ALONG LOW ACCELERATION TRAJECTORIES THROUGH THE INNER VAN ALLEN BELT

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INTRODUCTION

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The radiation hazard of energetic protons trapped in the Earth's magnetic field becomes an important consideration for manned, low-thrust, interplanetary vehicles because of the long period of time that such a vehicle must spend in the region near the Earth. The advantage of high payload ratios attainable with low-thrust electrically propelled vehicles may be seriously offset by the necessity of increased shield weight to protect against Van Allen radiation.

An estimate of the integrated dose received during a high thrust lunar trajectory through the radiation belts behind various thickness of carbon shield has been made (ref. 1). Also, a method of computing total time-integrated proton flux for an arbitrary trajectory through the inner Van Allen belt has been formulated, and examples of integrated flux for circular orbits are offered in reference 2. An approximate calculation of the upper and the lower limits of the integrated proton flux encountered during a low thrust departure from an Earth orbit was made for a thrust to weight ratio of 10^{-4} (ref. 3).

This paper presents an estimate of the time-integrated flux for trajectories having a constant acceleration in the range of 5×10^{-4} to

10^{-2} m/sec^2 and an inclination (angle between the vehicle orbit plane and the equatorial plane) in the range of 0° to 90°.

SPATIAL DISTRIBUTION OF FLUX

The distribution of the proton flux contours was assumed to be that given in reference 3 and is reproduced in figure 1. Flux contours of protons with energies greater than 40 Mev are plotted in the geomagnetic plane. The maximum intensity is 4×10^4 protons/(cm^2)(sec). This would be the actual distribution in space if the Earth's magnetic field were a perfect Earth-centered dipole field. This, of course, is not the case. The distribution is distorted to various degrees for different values of longitude. The variations in latitude and altitude of the base point in the figure are given in reference 3. It was assumed that every point in the distribution was rotated through the same angle and translated through the same distance as the base point for that particular longitude. This assumption implies that the central region of the distorted field can be represented by the central portion of a dipole field.

The approximation is better than the assumption that the Earth's magnetic field can be represented by a displaced dipole field but is inferior to a description of the field based on a spherical harmonic analysis. It is felt, however, that present-day uncertainties in the location of the upper altitude contours in the dipole plane and in the temporal variations of the contour locations do not warrant the computer time required for low-thrust calculations based on an accurate field description.

VEHICLE TRAJECTORY

The vehicle was assumed to have constant tangential acceleration throughout its passage through the belt. The results are also applicable with little error to vehicles employing constant-thrust engines since the change in mass during this period is quite small for the specific impulses typical of electric propulsion.

For a given flux distribution in space, the integrated flux incident on a vehicle is a function of the acceleration a, the inclination of the spiral plane to the equator I, and to some extent the coordinates of the point at which the spiral is initiated:

$$\text{integrated flux} = f(a, I, \text{starting point})$$

The integrated flux decreases with increasing acceleration because less time is spent in the region of the belts. There is a practical upper limit on the acceleration of vehicles employing electric engines that results from the substantial weight of the necessary electrical power-generation equipment. The effect of acceleration on integrated flux was investigated in the range of 5×10^{-4} to 10^{-2} m/sec^2.

For a given set of conditions, the integrated flux would be a minimum for an inclination of 90° (polar spiral). Other mission considerations, however, might make a smaller value of inclination desirable. The effect of inclination on the integrated flux was investigated throughout the range of possible inclinations.

The altitude of the initial circular-orbit radius has negligible effect on the integrated flux as long as it is less than the altitude of the bottom of the belt. A vehicle that initiates a spiral at a low altitude will eventually achieve the altitude of a vehicle that might start at a higher altitude. The differences in the trajectories from that point on are negligible. A best combination of longitude and latitude exists at which to initiate a spiral because of the radiation-free zones near the poles and the irregular shape of the belt about the Earth. For a large number of passes through the belt, as would be the case for a low-acceleration spiral, the advantage of starting at this point is negligible.

CALCULATION PROCEDURE

The integrated flux was calculated in terms of the equivalent number of hours a vehicle spends at the point of maximum intensity. It was felt that this measure of the flux would have greater physical significance than would the total number of protons striking a square centimeter of area. Equivalent hours, along with a set of dose rate curves appropriate for the heart of the belt, can be used to estimate total radiation dose. This procedure necessarily presumes that an invariant relative energy spectrum prevails throughout the belt, an assumption known to be untrue, but one that cannot be significantly improved upon with current information.

A short IBM 7094 computer program was written to compute the equivalent time at maximum intensity. The program accepts values of radius ratio ρ and central angle θ at various times t obtained from reference 4 for a particular acceleration. Values of ρ and θ are obtained at 2-min intervals by three-point interpolation. These polar coordinates in a plane inclined an angle I with the equator are transformed into geographic coordinates in a system that rotates with the Earth. Thus, the longitude, latitude, and altitude of the vehicle are obtained at each interval of time. The geographic coordinates are then corrected for the variation of the position of the flux contours with longitude. The corrected coordinates are used to obtain values of flux ratio (the ratio of flux at the point to the maximum flux) from a mathematical model of the distribution in the geomagnetic plane. The upper contours of the distribution were approximated by elliptical segments, the lower contours by segments of a circle. A smooth variation of intermediate contours was assumed. Finally, the values of flux ratio at 2-min intervals of time are numerically integrated by Simpson's rule. The result is the equivalent length of time spent at the point of maximum intensity expressed in hours.

RESULTS

The effect of acceleration can be seen in figure 2. Curves of equivalent hours at the heart of the belt versus acceleration are drawn for inclinations of 0°, 28.4°, 60°, and 90°. An orbit plane of 28.4° would be achieved by launching a rocket due east from Cape Canaveral.

The effect of inclination on the integrated flux is shown in figure 3. In this figure, the ratio of equivalent time for a particular inclination to the equivalent time for an equatorial spiral is plotted versus inclination. A considerable gain can be realized by increasing the inclination a small amount in the range of low inclinations. For a given acceleration, the integrated flux can be reduced by more than a factor of 4 by selection of a polar spiral rather than an equatorial spiral for a particular mission.

For accelerations not exceeding 10^{-2} m/sec^2 (above which the starting point of the trajectory becomes an important parameter), the data of figures 2 and 3 can be expressed by the simple relation

$$\text{equivalent time (hr)} = \frac{K_1 K_2}{a}$$

where

a acceleration, m/sec^2

K_1 0.25 (hr)(m)/sec^2, a constant obtained from figure 2

K_2 ratio of equivalent time for inclination I to equivalent time time for inclination of 0°, obtained from figure 3

As an example of the significance of the presented results, figure 2 indicates that a spacecraft traveling with an acceleration of 5×10^{-3} m/sec^2 in a plane inclined 28.4° would accumulate a dose equivalent to a residence of 28.5 hr in the heart of the proton belt. With the representative shielding data of reference 5, this would correspond to a required shield density of 100 g/cm^2 of aluminum if the dose were to be limited to 30 rem from this source.

REFERENCES

1. Barnes, T. G., Finkelman, E. M., and Barazotti, A. L.: Radiation Shielding of Lunar Spacecraft. Lunar Exploration and Spacecraft Systems, Plenum Press, 1962, pp. 52-71.

2. Perry, F. C.: Proton Fluxes Along Trajectories Through the Inner Van Allen Belt. Protection Against Radiation Hazards in Space, Book II, Proceeding of the Symposium at Gatlinburg, Tennessee, November 5-7, 1962, pp. 725-738.

3. Allen, R. I., et al.: Shielding Problems in Manned Space Vehicles. Lockheed Nuclear Products Report No. 140, 1961.

4. Moeckel, W. E.: Trajectories with Constant Tangential Thrust in Central Gravitational Fields. NASA TR R-53, 1960.

5. Beck, A. J., and Divita, E. L.: Evaluation of Space Radiation Doses Received within a Typical Spacecraft. ARS Jour., vol. 32, no. 11, Nov. 1962, pp. 1668-1676.

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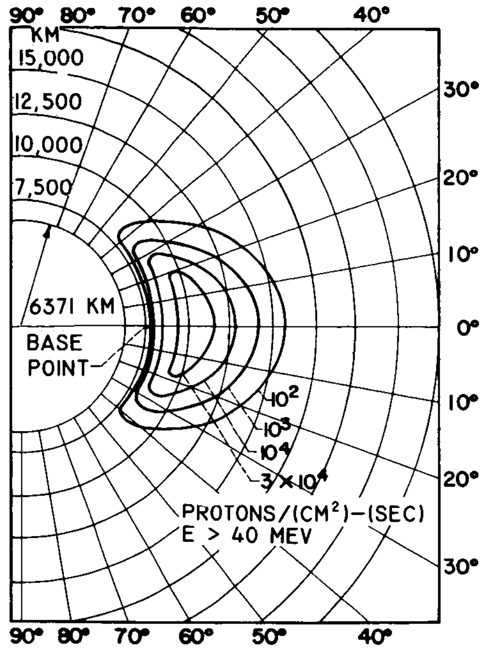

Fig. 1. - Proton distribution in the Van Allen belt plotted in the geomagnetic plane (reproduced from Ref. 3).

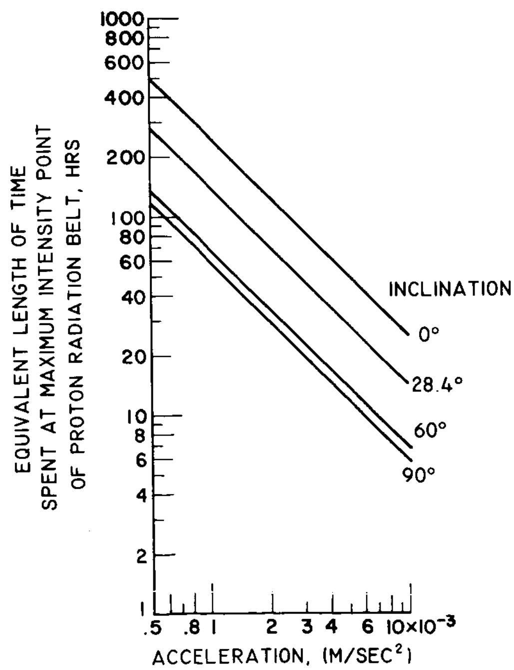

Fig. 2. - Effect of acceleration on integrated flux for various inclinations.

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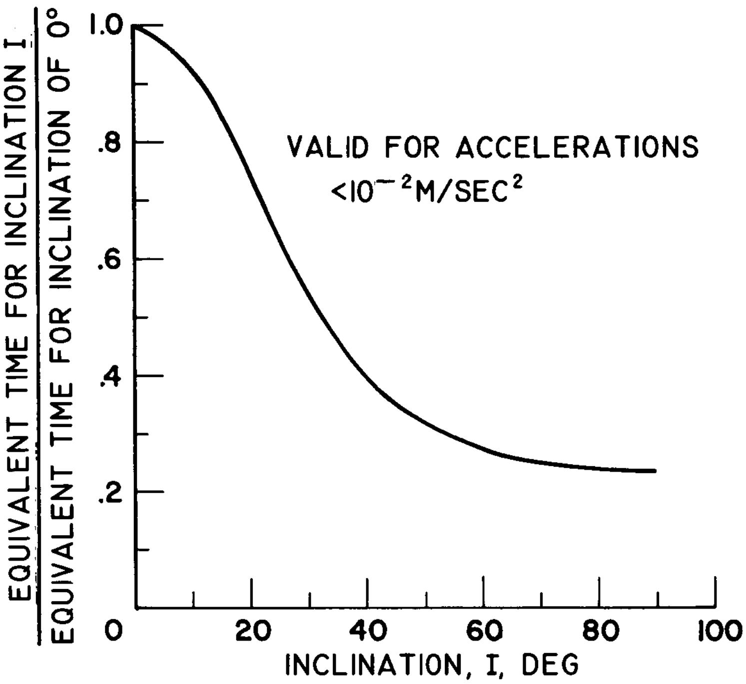

Fig. 3. - Effect of inclination on integrated flux.